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ABSTRACT

Presented are abstracts of 18 research reports. Topics covered include: (1) The effect of a numeration learning hierarchy on mathematics attitudes in kindergarten children; (2) Children's acquisition and production of mathematical rules; (3) Preschoolers' abilities to recognize counting errors; (4) Young children's solution processes for verbal addition and subtraction problems; (5) A system for diagnosing sixth graders' word problem difficulties; (6) Kindergartners' strategies for solving addition and subtraction problems; (7) Some implications for teaching problem solving; (8) Data on cognitive strategies of children with mathematical learning disorders; (9) Information about a project analyzing adolescents' problem-solving processes; (10) Young children's spatial preferences; (11) Assessing children's development in geometry using the Van Hiele levels; (12) Pupil concepts of ratio of distances in two dimensions; (13) Development of instruction designed to teach verbal problem solving in elementary mathematics; (14) Calculator use and problem-solving strategies of grade six pupils; (15) Effects of prior knowledge and creativity upon problem solving; (16) An analysis of selected cognitive style dimensions related to mathematics of college students; (17) Protocol analysis as a research tool in problem-solving studies; and (18) The effects of external structure and cognitive style on learning modulo fields. (MP)

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MATHEMATICS EDUCATION REPORTS

St. Louis, Missouri
22-25 April 1981

RESEARCH REPORTING SECTIONS

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

59TH ANNUAL MEETING

Jon L. Higgins, Editor

ERIC Clearinghouse for Science, Mathematics
and Environmental Education
The Ohio State University
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1200 Chambers Road, Third Floor
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FOREWORD

This Mathematics Education Report contains abstracts of all papers presented in the research reporting sessions of the 59th Annual Meeting of the National Council of Teachers of Mathematics. These papers were selected for presentation from a large number of proposals submitted to the NCTM Research Advisory Committee and the program chairman for Research Sections, Douglas A. Grouws. We wish to thank Dr. Grouws and the members of the Research Advisory Committee for making them available for this publication.

Jon L. Higgins
Research Associate
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59th Annual Meeting

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Reporting Session I

THE EFFECT OF A NUMERATION LEARNING HIERARCHY ON MATHEMATICS ATTITUDES IN KINDERGARTEN CHILDREN

Barbara Ann Wagner
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Barbara R. Sadowski
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Purpose: A positive attitude toward mathematics in the early school years is crucial to children's learning and progress in school. Some students find mathematics challenging and appealing; others avoid mathematics whenever possible. Feelings toward mathematics are important for educators to consider when faced with the task of helping children to learn about mathematics. The results of the Mathematics International Longitudinal Study (1972) indicated that attitudes toward mathematics are not as positive in their relation to achievement as are attitudes toward reading in six-year-old children.

This study examined the relationship between a numeration learning hierarchy curriculum design and student's perceived attitudes toward mathematics. The purpose of the study was to determine if kindergarten children's attitudes toward mathematics were significantly different for a group using a sequential, task-analyzed numeration learning hierarchy as the instructional mode in the classroom when compared to the attitudes of those children who are taught number concepts using curriculum not based on a numeration learning hierarchy. Inherent to this major research focus were two other questions concerning the reliability of measurement and the factors which underlie that attitudes toward mathematics in kindergarten children. Studies and current literature suggest that subject-related attitudes are formed early in children, that attitudes vary, and that attitudes are resistant to change.

The research hypothesis posited in this study was: Mean mathematics attitude scores will be significantly different in kindergarten students receiving instruction with a curriculum based on a numeration-learning hierarchy than the mean mathematics attitude scores of students not receiving instruction in a curriculum based on a numeration learning hierarchy. This entailed the adaptation and development of scale in a pilot study which led to the research instrument with a reliability of $r = .94$, that assessed the two dependent variables, Self-Affirmation of Mathematics and Fear of Mathematics in this study. The dual bases for the adaptation of the instrument were the constructs of the Revised Aiken-Dreger Attitude toward Mathematics Scale (1968) and the concrete response modality of the Student Evaluation of Teachers Instrument by Hoak, Kleiber, and Peck (1972).

Procedure: The experimental study involved four instructional weeks (15 minutes each day for 5 days) for kindergarten students using the numeration learning hierarchy curriculum, Fundamentals Underlying Number, by Wilson and Uprichard (1972). The organizational scheme of the treatment curriculum had these major aspects: (1) the order in which major concepts were presented; (2) the progression of levels within each concept (3) the spiraling of levels and concepts in stages; (4) the use of concrete, manipulative materials in game formats; (5) self-checking processes; and (6) active student involvement and participation. The control group continued to receive the usual instruction in arithmetic.

The teachers and kindergarten students participating in this study were drawn from a suburban school district in southwestern United States. Kindergarten subjects were utilized from both morning and afternoon classes. Within four schools, 12 kindergarten teachers were randomly assigned to six morning and six afternoon experimental groups and six morning and six afternoon control groups. From 404 kindergarten students in the study, a final sample of 120 experimental and 120 control students was randomly selected for the posttest-only collection of data. A nested posttest-only design was utilized to minimize the confounding effects of different teachers, methods of personal interaction in the classroom, and times of day.

Results: The null hypothesis was tested using multivariate analysis of variance. An overall significant difference between the experimental group and control group was indicated by an omnibus F with 2 and 237 degrees of freedom, equal to 55.10 ($F_{2,237} = 55.10$; $p < .0001$. This data provided an affirmative answer to the major research question of the study. A curriculum based on a hierarchy of numeration concepts which emphasizes student confirmation of responses and where the rate of progressing through the curriculum is determined by student's number concept development results in more positive attitudes toward mathematics in kindergarten children. An anlysis of the items on the scales further showed that experimental subjects had significantly less fear of number, significantly greater confidence in their ability to handle numbers, and were significantly more positive in their enjoyment of mathematics.

Reporting Session I

CHILDREN'S ACQUISITION AND PRODUCTION OF FUNCTIONAL RULES

Merlyn Behr
and
Helen Adi
Northern Illinois University

Purpose: To discover a general mathematical rule (concept, pattern, or function) from a given set of data and to demonstrate that the rule was discovered constitute what is known as inferencing behavior. The purpose of the present study was to investigate children's mathematical inferencing behavior.

Concept-attainment research, and information processing research have investigated inferencing behavior from two different points of view. Concept attainment research focuses on the student's ability to recognize given information as positive or negative instances of a concept or a rule. Whereas, information-processing research focuses on the student's ability to generate a rule description from a given set of positive instances and to use the induced rule description to produce other positive instances of the rule (to extrapolate). Many times a rule may be attained by the student but he/she may be unable to describe it at the verbal level. In the present study, we explore the students' mathematical inferencing behavior by observing his/her ability to generate hypotheses, to produce positive instances of the induced rule, and to describe the rule.

Wason and Johnson-Laird (1972), investigated the inferencing behavior of adults on "concept-description" tasks. Two major reasoning components were identified: (1) the ability to generate hypotheses from given data, and (2) the ability to relinquish hypotheses in the presence of further contradictory evidence. How would the child's inferencing behavior be in comparison to the adult's? Would the child accept to relinquish hypothesized rules in face of contrary evidence?

Does the child infer the hypotheses from the given data, or does he/she select a tentative hypothesis from his/her own mental repertoire, and then tries to fit this hypothesis onto the given data? Does the process of discovery in the child involve more of hypothesis modification or of hypothesis selection of an available set of alternates which is already in memory? What observations of the data are more likely to induce correct rule attainment and production? What kinds of errors are more likely to occur among children during this inference? Faced with contrary evidence, does the child always realize that her/his hypothesized rule was faulty?

The focus of the present study is more on the processes used to attain and to produce mathematical rules from given sets of information, than on the mere ability (or inability) of children to discover these rules.

In a study on human acquisition of concepts for sequential patterns, Simon and Kotovsky (1963, 1973) reported that subjects first discover the pattern, then construct a mental description of the pattern and finally use the pattern description to make an extrapolation. But to use the pattern description it is assumed that the subject possesses a pattern generator which applies on the stored pattern description. What perceptual and organizational components of the task would induce clearer pattern descriptions, and easier rule productions? How many exemplars of the rule need to be provided for the construction of pattern or rule descriptions? What does the child bring with him to the task? For example, to discover informally that $y=x^2$ describes the order pairs {(1,1), (2, 4)} what mathematical competency, or level of sequential reasoning ability does the student need to have? What perceptual presentational modes of the given information would better induce the correct functional rule? In how many different ways would children describe the same mathematical rule?

The present study based on concept-attainment, and information-processing paradigms, explored children's (3rd and 5th graders) inferencing behavior in terms of how they try to discover (attain, extrapolate, and describe) mathematical functional rules from differently presented sets of ordered pairs of numbers.

Procedure: Fifty elementary school children equally divided among third and fifth graders were randomly selected to participate in the study. All students were administered two tests on the individual difference variable of sequential reasoning ability: (1) Shipley's Series Completion Test, and (2) Raven's Progressive Matrices Test.

Five different mathematical rules were selected for rule-attainment-production tasks: (1) $y = x + 1$, (2) $y = 2x$, (3) $y = 3x - 2$, (4) $y = x^2$, and (5) $y < 2x$. Positive instances of ordered pairs of the rule were presented for each task. Five presentational modes were used: (1) graphic, graphic (g,g), (2) symbolic, symbolic (s,s), (3) pictorial, symbolic (p,s), (4) symbolic, graphic (s,g), and (5) pictorial, pictorial (p,p). Only one of these modes was within one task. A pictorial, symbolic presentational mode, for example, of [illegible]en ordered pair of a function, means that the abscissa of the ordered pair was presented pictorially whereas the corresponding ordinate was presented symbolically.

The five functional rules were randomly sequenced, and a Latin Square design was used to generate five different sets of problems as follows:

	$y = x + 1$	$y = 2x$	$y = 3x - 2$	$y = x^2$	$y < 2x$
Group 1:	gg	ss	ps	sg	pp
Group 2:	ss	ps	sg	pp	gg
Group 3:	ps	sg	pp	gg	ss
Group 4:	sg	pp	gg	ss	ps
Group 5:	pp	gg	ss	ps	sg

Each student was randomly assigned to one of the five groups of experimental tasks. The five tasks were randomly sequenced for each student, and were presented in a one-to-one interview setting. All interviews were audio recorded, and the interview duration times ranged from 20 to 60 minutes per interview. During the interview only the presentation of the tasks was structured, probing into the student's reasoning and processing mechanisms was encouraged. Two persons (present investigators) conducted the interviews. All tasks and interview protocols were piloted prior to the study.

For each functional rule, two positive instances were presented on a page, the abscissa of the 3rd instance was given and students were asked to find the pattern from the first two instances and to guess and write the ordinate of the 3rd instance. Feedback was provided and the correct ordinate was given in case it was wrong. A fourth ordered pair of the function was given, and the abscissa of the 5th instance. Students were then asked to guess and write the value of the corresponding ordinate (by now they had seen four positive instances of the rule). This procedure was repeated until students were asked to produce the ordinate of the 8th ordered pair. Then, they were asked to verbalize the functional rule and to explain how they had attained it. At this point no feedback was provided.

After the description of the rule students were asked to produce an ordered pair that fits in with the data and with their described version of the rule. They were also asked to express the functional rule in writing.

Analysis: In the analysis, the effects of sequential reasoning ability and different presentational modes of information on students' performance will be considered. For each of the mathematical rules, answers to the following questions will be provided:

(1) Did Ss guess the rule? After how many positive instances? What types of hypothesized rules were given? Any differences between grade levels?

(2) To what extent were Ss persistant of their hypothesized rules? Did Ss maintain the same rule even in the face of contrary evidence?

(3) Do Ss use the information of the first ordered pair?

(4) Do Ss use the same rule or change the rule for increasing abscissa values?

(5) What perceptual and content features of the given ordered pairs do Ss focus on to generate the rule description?

(6) How does the verbal expression of the rule compare with the written expression of the rule?

Student protocols are currently being analyzed. A complete description of the results will be made available at the NCTM meeting.

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Reporting Session I

PRESCHOOLERS' ABILITIES TO RECOGNIZE COUNTING ERRORS

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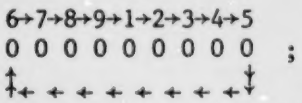
Purpose: Traditionally, the behavior of pre-schoolers has been characterized in terms of the skills and knowledge that they lack when compared to older children. Recently, however, researchers have started to focus on the capabilities preschoolers do have (Gelman, 1978; Siegler, 1980). Counting is an ideal context for investigating young children's abilities and how they develop. Even young children are not totally incapable; in fact, most of them are surprisingly good counters at least within some contexts, such as counting small sets of objects (Gelman & Gallistel, 1978; Fuson & Mierkiewicz, 1980). Yet, there are marked differences between the counting skills of younger and older children, so developmental changes in capabilities are observable. In addition, counting itself is an important skill for investigation because it is a prerequisite for many arithmetic concepts as well as being part of everyday life.

A major question is what kind of underlying knowledge generates children's counting performance. One claim is that this performance stems from children's knowledge of a set of counting principles (Gelman & Gallistel, 1978). However, this was inferred primarily from tasks children performed correctly. Thus, it is difficult to know if they understood the underlying principles or just knew how to execute the procedure correctly. In addition, performance that did not conform to these principles was attributed to difficulties in executing the procedure rather than a lack of knowledge of the principles. The validity of this claim could not be easily assessed. Data from spontaneous performance alone is not adequate for distinguishing whether counting is based on a knowledge of principles of simply on knowledge about the procedure.

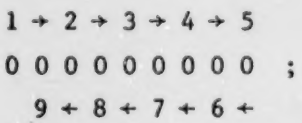
A different prospective on this issue is provided by examining children's recognition of counting mistakes as errors and to distinguish these mistakes from unconventional but still correct counting. This study examines three-, four-, and five-year-olds' abilities to make these distinctions with respect to the one-to-one correspondence between words and objects when counting rows of fixed items.

Four principle types of errors were used in this study: omitted word (an object pointed to without a word), extra word (an object with two words and one point), skipped object (an object for which no word or point was given), and doubly counted object (assigning two successive word-point pairs to one object). Each of these errors could occur in the beginning, middle, or end of a row. In addition, each type and location of error was examined in a context of short (3 or 4 item) or long (9 or 10 item) rows.

Four types of unconventional counting that maintained the one-to-one word-object correspondence were also included: (1) counting in the opposite direction from that of the child: (2) starting to count in the middle of a row, e.g.



(3) counting alternate objects, e.g.



and (4) double pointing to an object while saying only one word--not a counting error because the one-to-one word-object correspondence is maintained. Standard correct counting trials were also included.

Procedure: Thirty three-, four-, and five-year olds from a predominatly middle class pre-school participated in the study. Age groups were balanced by sex and order of presentation of the count trials. Rows of 3,4,9 and 10 plastic chips spaced approximately an inch apart on cardboard strips were used. Chips within each row alternated in color (red or green) to maximize the children's ability to distinguish adjacent chips.

During the experimental sessions, a child watched a puppet count a series of rows of chips. When the puppet finished counting each row, the child was asked if the puppet had made a mistake, and if so, what he had done wrong.

Results: Of major interest were possible developmental changes in children's abilities to detect counting errors and to distinguish them from unconventional but correct and standard correct counting; the effect of set size on this ability; children's recognition of different types of errors and the related effects of location of the error and set size; and the relationship between the type of true errors a child identified and the type of errors he made in his own counting. Due to space limitations, only some of these results will be briefly summarized here with more detailed results reported in the complete paper.

Results indicated that (1) children in all age groups were very good at recognizing standard correct counting as correct; (2) in general, children were poorest at recognizing unconventional but correct counting;

and (3) ability to detect true counting errors increased with age, with the three-year-olds only consistently identifying skipped object errors, while the five-year-olds identified all errors more than 90% of the time. Error recognition was significantly better on small than on large sets, as was detection of errors made at the end of a count compared to those in the beginning or middle. A positive relationship was found between a child's ability to detect counting errors and accuracy of his own counting.

Conclusions: These data suggest that children's knowledge about counting initially develops in a limited, piecemeal fashion. One of the earliest criteria for correct counting appears to be that it is wrong to totally skip an object. The point or say a word "only once" aspect does not appear as a criteria for correct counting until later. Even then, older children's difficulties with correct but unconventional counting seem to suggest that although they may know that one-to-one correspondence is necessary for correct counting, they do not realize it is sufficient. Thus the knowledge underlying counting performance may be better characterized as a collection of specific rules for executing the standard procedure rather than as a few general principles. The following developmental sequence is hypothesized: children first inductively acquire specific rules (e.g. don't skip objects); then consolidate some of these specific rules into the more general rule of one-to-one correspondence, but also consider other features as necessary for correct counting as well (e.g., start counting at an end); and finally realize the sufficiency of one-to-one correspondence. Additional results supporting this progression will be presented.

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Reporting Session II

YOUNG CHILDREN'S SOLUTION PROCESSES FOR VERBAL ADDITION AND SUBTRACTION PROBLEMS: THE EFFECT OF THE POSITION OF THE UNKNOWN SET

James Hiebert
University of Kentucky

Purpose: It is now well documented that young children can solve a variety of addition and subtraction story problems, even before they receive formal arithmetic instruction. It is also well known that different story situations elicit different solution strategies and, apparently, different arithmetical interpretations. It is less clear what factors account for these differences. Recent attention in this area of research has been directed toward an identification of task variables which influence children's interpretation of verbal problems and affect the relative difficulty of various problem types. Three factors consistently emerge as being especially important: semantic structure of the problem; number size; and position of the unknown quantity in the associated number sentence. The major purpose of this study was to systematically examine the effect of the third of these factors, the position of the unknown set, on first-grade children's representation and solution processes for verbally presented addition and subtraction problems.

Recent research has shown that the semantic structure of a verbal problem is a major determinant of task difficulty, and of the type of strategy which young children use to solve the problem. It appears that, at least before receiving instruction, children solve verbal problems by representing the problem directly and carrying out the prescribed action.

The hypthesis that young children rely on direct representation to model and solve problems suggests that problems which are not directly representable would elicit different solution strategies and would be more difficult to solve. These are problems in which the first quantity described in the problem is the unknown. This means that the position of the unknown in the associated number sentence may be an important factor in children's interpretation of the verbal problem and a major determinant of problem success. Although this question has been considered with older children using symbolic number sentences, it has not been systematically investigated with young children using verbal problems.

Procedure: The subjects were 47 first-grade children attending one of the public schools in a moderate-size midwestern city. At the time of testing, in March, the children had received no formal instruction in solving verbal problems.

The arithmetic problems used in the study were three joining (addition with described action) problems and three separating (subtraction with described action) problems. The problems were generated by placing the unknown in each of the three positions in the associated number sentence. A sample joining problem with an associated number sentence of a +□ = c is: "Sally had "a" books. Her brother gave her some more books. Now she has "c" books altogether. How many books did Sally's brother give her?" Parallel phrasing was used in the other five problem-types. Each problem type was presented using small numbers (sum between 6 and 9) and large numbers (sum between 12 and 14).

The 12 problems were read to each subject in an individual interview. Small cubes were available and the subjects were told they could use the cubes to help them solve the problems. The six small number problems were presented first. If a child initiated a reasonable solution strategy for at least one of these problems he/she was then given the large number problems, otherwise the interview was terminated.

Results: The results of primary interest are the representations used to model the problems and the strategies used to solve them. The position of the unknown had a substantial effect on children's modeling behavior. With respect to the small number problems, 55% of the responses to the verbal representations of both a + b =□ and a - b =□ included representing the sets with cubes or fingers. This percentage drops to 44% for the a ±□ = c problems and 19% for the □ ± b = c problems. Similar patterns were evident for the large number problems as well. These percentages are also accurate indicators of the relative difficulty of the various problem types. The similarity of behavior on a given pair of addition/subtraction problems reinforces the apparent influence of the position of the inknown in children's ability to model the sets and solve the problem.

In general, the strategies used to solve a problem matched its semantic structure. For example, the strategy used most often to solve a - b =□ was to represent the set "a" with cubes, remove "b" cubes, and count the remaining cubes. In contract, the dominant solution strategy for a -□ = c was to model set "a", remove cubes until "c" cubes were left, and then count those removed. This slight but significant adjustment in the separating procedure is consistent with the semantic structure of the problem. Direct solution of the □ ± b = c problems involves trial and error strategies. Although a total of only seven such strategies were observed, they occurred exclusively on these two problem-types. This accounted for 35% of the appropriate counting strategies used on these problems.

Conclusions: These results strongly support the hypothesis that, before receiving instruction, young children solve verbal problems by modeling the sets and operating directly on these representations. Furthermore, the results suggest that if a problem is amenable to direct representation it is more likely to be interpreted and solved correctly. Consequently, the position of the unknown is an important determinant of verbal success. These results have important implications for instruction since it may be that the initial development of meaningful addition and subtraction concepts is best supported by the use of directly representable verbal problems.

DIAGNOSING SIXTH GRADERS' WORD PROBLEM DIFFICULTIES

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and
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Purpose: The purpose of this study was to develop and field-test a systematic technique for diagnosing difficulties in solving word problems. The investigators are attempting to develop a system which classroom teachers can use to determine the main source of difficulty for each child in solving word problems. Specifically, the diagnostic system is designed to answer these two questions for each child:

1. At what level does this child now solve word problems satisfactorily?

2. What is the main source of difficulty preventing this child from solving word problems satisfactorily at the next highest level?

Word problems are frequently mentioned as presenting great difficulties for many students. Shiela Tobias (1978) points out that unsuccessful encounters with word problems in the elementary school may well mark the beginning of "math anxiety" for many people.

The recognized difficulty and the practical importance of word problems provide two good reasons for developing a system of diagnosing the difficulties of individual students. A third reason derives from the "back-to-basics" movement of recent years. There has been for some time a deepening public concern for the teaching of basic skills. This public concern is manifested by the increasing use of required minimal competency tests and by the every-pupil tests required at specific grade levels. The danger from this concern and the resulting tests is that "basic mathematical skills" may come to be considered as computational skills, omitting other important skill areas such as problem solving. The National Council of Teachers of Mathematics, in an official position statement (1978), urges teachers of mathematics to respond to this concern in positive ways, they need verified knowledge about the best ways to go about teaching students basic skills and about what causes the major difficulties students have.

The National Council of Supervisors of Mathematics (1978) takes the position that too narrow an interpretation of basic skills tends to de-emphasize the

importance of teaching for mathematical understanding. Word problems provide one avenue for the development of understanding while at the same time providing the opportunity to develop computational skills.

Procedure: This design is based on the concept that solving word problems requires four different abilities each of which may either be seen as a holistic ability or as a cluster of skills. These four abilities are: (1) the ability to read the problem, (2) the ability to set up the problem so that the necessary computation is ready to be performed, (3) the ability to perform the necessary computation, and (4) the ability to integrate reading, problem interpretation, and computation. Once word-problem solving is defined to require these four abilities, certain questions arise. The main research questions of interest to this study were:

1. What proportion of students have as their main source of difficulty with word problems each of the following factors?
   a. Computational skills
   b. Problem interpretation
   c. Reading
   d. Integrating these skills into the solution of a problem

2. Can students be diagnosed as having a major need in one of the four categories?

The answer to these questions have relevance for the diagnosis, teaching, and evaluation of basic mathematical skills. A basal mathematics series for use in grades 3-8 but which was not in use in North Carolina schools was used to construct a series of graded tests. Three tests were developed at each level. A test of computation was constructed for each grade level of the series by randomly selecting one of the first three word problems from each section of the book which contained word problems. Each of these word problems was set up in pure computational form so that this test would only be testing the computation skills which the problems require. All reading and problem interpretation tasks as well as the necessity for integration, were removed by virtue of the problems being set up in pure computational form.

A problem-interpretation test was constructed for each level of the series by randomly selecting a second of the first three word problems from each section of the book having word problems. These problems were read to the students and two scores were given based on whether or not the students set them up properly and on whether the answer was correct.

A third test was constructed for each level by using the remaining problem from the first three word problems from each section of the book. This third test also yielded two scores: one computed by grading the students on whether or not they could read the problems independently and set them up properly, and the other be grading to see if they achieved the correct answers after reading them independently. The test is the same; the two scores are derived by using two different grading systems for the one test.

These three tests were administered to each student from each level of the series until the student could be given a profile consisting of five scores. A student's computation level was the highest grade level in the series where the student could correctly work 75% of the problems when they were set up for him or her. A student's problem interpretation level was the highest grade level where the student could correctly set up 75% of the problems when they were read to him or her. A student's listening-problem solving level was the highest grade level where the student could correctly solve 75% of the problems when they were read to him or her. A student's reading level was the highest grade level where the student could correctly set up 75% of the problems without having the problems read for him or her. A student's reading-problem solving level was the highest grade level where the student could correctly solve 75% of the problems without having the problems read for him or her. These five scores constitute a diagnostic profile for a student.

Tests constructed and scored according to the above criteria were administered to 244 sixth-grade students in two different schools. Complete data were obtained from 217 of these students.

Results: The first research question inquired into whether the difficulties of the majority of students can be placed in any one of the four main sources. The following table shows the percentages of students in this study falling into each category:

Main Source of Difficulty	Percentage of Students
Computation	26%
Reading	29%
Problem Interpretation	19%
Integration of the Three Skills into Total Problem Solving	26%

The second research question asked whether individual students could be diagnosed as having a major need in one of the four areas. Data produced by the study show that a diagnostic profile identifying the major need can be constructed in almost all cases.

Conclusion: A systematic diagnostic procedure, which classroom teachers can devise for themselves and use, has been developed and tested with over 200 sixth-grade students. Data from this study show that word-problem difficulties cannot all be treated the same way with different children.

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Reporting Session II

KINDERGARTNERS' STRATEGIES FOR SOLVING ADDITION AND SUBTRACTION PROBLEMS

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Purpose: Although data exist concerning kindergartners' ability to solve simple addition and subtraction problems (e.g., Ibarra & Lindvall, 1979; Riley, 1979) this research has often focused on correct answers and provides little insight into the problem solving processes which children exhibit prior to formal instruction on addition and subtraction. The purpose of this study was to describe in detail two aspects of the strategies which kindergartners use to solve addition and subtraction problems. One aspect involved describing the problem solving strategies which kindergartners develop prior to instruction on these operations. The particular strategies of interest were "heuristic strategies" (Carpenter, 1980) in which known number facts are used to derive other number facts needed for solution of the problem, e.g., for 7 + 4 = ?, one heuristic strategy would be "7 + 3 = 10, so 7 + 4 = 11." The use of such strategies prior to instruction indicates that children not only spontaneously develop problem solving strategies independent of formal instruction, but that in a simple setting young children are able to employ the heuristic of using the solution of a related or simpler problem (Polya, 1957) in the problem solving process. Several studies (Ilg & Ames, 1951; Hebbeler, 1977) have identified some of the strategies which kindergartners use on addition and subtraction problems but have not systematically documented their use of heuristic strategies.

Children's errors and inappropriate strategies were the second aspect of kindergartners' processes which was addressed by this study. Lindvall and Ibarra (1980) have discussed some of the difficulties kindergartners have in modeling addition and subtraction problems but no systematic and comprehensive description of kindergartners' errors and misconceptions exists. Such information is necessary to describe individual differences in children's approaches to solving addition and subtraction problems and their readiness for formal instruction on these operations.

Heuristic strategies are among the most sophisticated strategies available to children for solving addition and subtraction problems. It is important for teachers and curriculum developers to be aware of the extent to which such strategies are used by children prior to instruction on the operations of addition and subtraction. Likewise it is important for teachers to be aware of children's errors and the misconceptions which are evident from the use of inappropraite strategies.

Procedures: The sample for the study consisted of 50 mid-year kindergartners from two small-town/rural schools. Subjects were administered two individual interviews during which they solved 12 verbal addition and subtraction problems and 12 parallel abstract problems with sums less than 10. Manipulatives were available on all problems and subjects' modeling procedures, solution strategies, and errors were observed and recorded.

Data from the study were analyzed descriptively. Heuristic strategies were first categorized along two dimensions, the source of the numbers used when deriving the needed number fact and the method of deriving that fact. The requency with which kindergartners used heuristic strategies was then described for each of these demensions. Use of heuristic strategies was also described for the abstract and verbal problem presentation contexts.

Kindergartners' use of inappropriate strategies was documented by classifying errors into procedural errors and errors of interpretation were of primary interest and were further partitioned into types: Incorrect interpretation of the problem (use of the wrong operation), Incorrect modeling of an operation, Identification of the answer, and Superficial solution (guessing, no attempt, responding with a number given in the problem, etc.). The frequency and instances of each of these errors were then documented within the 1200 items administered to the sample.

Results: Heuristic strategies were used at least once by 22% of the kindergartners interviewed. This suggests that many kindergartners possess the capability to apply such a strategy to simple addition and subtraction problems prior to receiving instruction thereon. The types of heuristics used varied widely but could be described primarily as ones in which the numbers used involved doubles or other known facts. Along the dimension of the method used for deriving the desired fact approximately three-fourths of the heuristics involved a compensation strategy such as "3 + 3 is 6, so 2 + 4 must be 6" rather than a decomposition strategy such as "3 + 3 is 6, so 3 + 5 must be 8." Heuristics were used approximately twice as often on abstract problems as on verbal problems.

Only one-fourth of kindergartners' errors were procedural errors in which a correct strategy was applied to the problem but miscounting or forgetting occurred. A majority of the errors were errors of interpretation; approximately half of these involved application of a systematic but erroneous strategy. Superficial solutions accounted for three-fourths of the interpretation errors, while incorrect interpretation, incorrect modeling and identification of the answer accounted for 16%, 3%, and 6% of the errors of interpretation, respectively. Of particular interest were the modeling errors which occurred on subtraction problems. These included attempts to model subtraction similarly to addition in which two sets were formed initially and one was then removed as well as attempts to somehow remove two sets from the entire supply of available manipulatives. These errors, in particular, indicated misconceptions about modeling the operation of subtraction and have implications for diagnostic work in the classroom.

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Reporting Session III

SOME IMPLICATIONS FOR THE TEACHING OF ARITHMETIC STORY PROBLEM SOLVING DERIVED FROM RESEARCH IN THE COGNITIVE SCIENCES

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Developments in the area of theory and research on human problem solving that have taken place in recent years appear to have definite implications for what is involved in the understanding and solving of arithmetic story problems and, hence, for the teaching of these capabilities. These developments have been the product of the work of cognitive psychologists, computer scientists, and other researchers, including mathematicians and mathematics educators, who have focused on the cognitive abilities essential in such problem solving. The presentation abstracted here will attempt to summarize some of the major results of the efforts of such persons, will outline a set of principles that appear to be supported by their work, and provide some specific examples of materials and procedures that should be of value to elementary school teachers in teaching pupils how to solve arithmetic story problems.

The work of elementary school children in solving arithmetic story problems should always represent a true problem solving activity. As such, it should include a careful analysis of the story and of the relationships or actions involved as steps to be completed before an arithmetic operation is selected and carried out. That is, an emphasis must be placed on pupil understanding of the problem if what is learned in school is to be of value to the students in solving practical problems in real life. Some recent work of cognitive psychologists has served to clarify the nature of understanding and of what is involved in human problem solving (Larkin, Heller & Greeno, in press). The efforts of a number of persons in developing computer simulations of what takes place in a variety of problem solving situations has done much to indicate what types of procedures may be employed by effective problem solvers and also the knowledge structures that are essential to the use of such procedures. In addition, a number of researchers from a variety of disciplines have used clinical interview techniques and extensive observation procedures to identify what human subjects actually do in solving a number of types of problems, including arithmetic story problems. The following set of generalizations has been gleaned from a review of some of this recent research and is proposed here as a partial list of a set of principles that should be useful in planning instruction on solving arithmetic story problems. For most of the principles listed, sources which appear to support it are cited.

Some Suggested Principles for the Teaching of Story Problem Solving

1. At all elementary school grade levels, beginning with kindergarten, the solving of story problems should be taught as a true problem solving experience, involving a clear understanding of the story and of why certain arithmetic operations should be applied. The solving of story problems represents the application of a powerful tool, the development of a "Mathematical model" to solve a practical problem. It merits attention and emphasis on a continuing basis. (Skemp, 1971; Strauch 1976).

2. The development of a "Mathematical model" (e.g., a number sentence, an operation, an equation) for a problem should be taught as involving two steps; (a) the development of an intermediate representation (a Physical model, a diagram, table, etc.) by abstracting the essential information from the story and (b) then using this intermediate representation to identify the necessary mathematical representation. (Newell and Simon, 1972; Greeno, 1977; Strauch, 1976).

3. Most pre-shcool and primary grade children display considerable understanding of simple quantitative problems when they use manipulative materials in demonstrating the solution process. (Carpenter, Hiebert, & Moser, 1979; Gelman & Gallistel, 1978; Hebbeler, 1977; Ibarra, 1979). This type of understanding, based on a physical representation of problem situations and solution, should be a definite goal of instruction as children work with more complex problems at higher grade levels. (Greeno, 1977).

4. The development of intermediate representations or physical models will be a difficult task for many students, particularly with certain types of stories (Heller, 1980; Lindvall & Ibarra, 1980). The ease with which a few students in a class may be able to do this should not blind the teacher to the difficulty that it poses for many students (Ibarra, 1979).

5. Useful intermediate representations can take many forms, but, whatever its form, any useful representation must contain all relevant components of the problem and must permit the exemplification of any described relationships and operations (Skemp, 1971).

6. The effective use of a model is dependent upon the availability of certain knowledge or knowledge structures that must be employed in comprehending the story and in manipulating the model. (Heller, 1980; Larkin, Heller, & Greeno, in press; Riley & Robinson, 1980).

7. Procedures for developing models of stories or for using other methods of analysis must be taught systematically, with attention given to models for each type of story and to steps for determining when to employ a given type of model (Schoenfeld, 1978; Carpenter & Moser, 1979).

8. Using a "key word" approach to story problem solution does not promote understanding (Hinsely, Hayes, & Simon, 1976). Comprehension of common terms must be taught, but they should be taught in relationship to what they imply for 'modelling with understanding.'

9. Speed should not be emphasized in the solution of story problems. If understanding is to be a major goal, students must be given time to develop that understanding. An emphasis on speed encourages students to guess at what operation to use and then proceed without really understanding the story.

Reporting Session IV

RESULTS OF RESEARCH BY THE ARITHMETIC TASK FORCE ON COMPUTATIONAL SKILLS, WORD PROBLEM SOLVING SKILLS, AND RELATED COGNITIVE STRATEGIES OF CHILDREN WITH ARITHMETIC LEARNING DISORDERS

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and
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Arithmetic disorders in children with learning disabilities have received relatively little attention. The Arithmetic Task Force of the Research Institute for the Study of Learning Disabilities, Teachers College, Columbia University, was created to address the need for a clearer understanding of the nature of arithmetic disorders. It has been engaged in a comprehensive investigation of arithmetic learning disorders for the past three years.

The work of the Institute is predicated on a postulation of information processing differences or deficits as an operational definition of learning disability. Chief among these is an inability to develop and use a set of cognitive strategies to formulate and carry out an habituated cognitive plan. The effect of this postulated deficit on arithmetic learning would be vast, for the essence of arithmetic processing is the mastery of and use of a variety of algorithms.

The research to date has focused on:

- generation of data which are descriptive of the computational and problem solving performance of learning disabled and nondisabled children.
- investigation of teaching methods designed to improve the arithmetic computation and problem solving performance of children with arithmetic learning disorders.
- development of a method of screening young children who are at risk for the development of arithmetic learning disorders.
- investigation of the relationship of cognitive styles and study strategies of children with arithmetic learning disorders to proficiency in arithmetic.

The results of these studies and their implications for classroom instruction will be presented by a panel of investigators utilizing videotape, overhead projection, and slides. Sample instructional materials will be available for examination.

The initial studies compared learning disabled children and their nondisabled peers in speed and accuracy of performance on tests of basic fact proficiency. Subjects for the study included 183 learning disabled students and 842 nondisabled children in two school districts (one rural and the other rural-suburban).

Each of the subjects was given a group administered test of proficiency in addition and subtraction basic facts. Fifth and sixth graders completed an additional test of multiplication facts.

Analysis of variance on data for each operation revealed significant main effects for age and group, with nondisabled students' performance higher than the learning disabled at each age level. Examination of test protocols indicated the use of an overt counting procedure to arrive at the solution by a substantial number of learning disabled children.

Two follow-up studies investigated the efficacy of two methods of basic fact instruction with learning disabled subjects. Method A utilized the traditional instructional sequence for addition or multiplication facts. Method B, based on the work of Carol Thornton, stressed the use of cognitive strategies to generate groups of related number facts.

Common characteristics of both instructional methods were: carefully sequenced set of lessons; daily monitoring of individual pupil progress; and use of a highly motivating game format to provide practice. The findings of these studies demonstrated that significant group mean gains in speed and accuracy performance on tests of basic facts were achieved under each method, suggesting that gains were related to the framework of the program and not to the order in which clusters of facts are presented.

Another line of investigation concerned the relationship between visual-perceptual ability and arithmetic achievement. In an initial investigation, 46 learning disabled and 97 nondisabled boys in grades 4-6 were given a battery of seven visual perceptual tests, the arithmetic subtests of the Peabody Individual Achievement Test (PIAT), and the Wide Range Achievement Test (WRAT).

Results of stepwise multiple regression analysis revealed three of the four variables which accounted for 26.5 percent of the variance in learning disabled performance on the WRAT were measures of visual perception. The contribution of visual-perceptual measures to PIAT performance was considerably less, but also significant. The predictor variables of IQ and visual perception also correlated significantly with arithmetic performance within the normal sample.

It was concluded that visual perception is a probable good predictor of arithmetic achievement. This study provides support for the notion that perceptual functioning is related to overall arithmetic achievement in learning disabled children. Since further investigation in this area seemed amply justified, current research efforts have involved the development of a battery of tests to identify kindergarten and first grade children at risk for potential arithmetic learning disorders. Screening tests were administered to 215 kindergarten children in a New York suburb and to 1,200 kindergarten and first grade children in the commonwealth of Puerto Rico. One year achievement measures were collected and longitudinal data will be collected for the next several years. Using multiple regression analysis, the factors most predictive of arithmetic achievement in the primary years will be identified.

The final group of studies focuses on the word-problem solving performance of learning disabled and nondisabled children. Videotapes were prepared of learning disabled and normal children solving a variety of verbal problems. Among the difficulties displayed by learning disabled children, as analyzed by experienced teachers and clinicians, were: inadequate mastery of computation; lack of familiarity with the context of the problem; inability to distinguish relevant from irrelevant information and/or inability to disregard irrelevant data; inability to identify the operation required to solve the problem; lack of knowledge of vocabulary of problems; and failure to perceive multiple steps required in the problems.

Analysis of a study comparing the word problem solving ability of disabled and nondisabled students revealed significant main effects for age and group with the nondisabled students' performance higher than the learning disabled at each age level.

Initial investigation into word problem solving strategies suggests that there is a relationship between cognitive style and the ability to solve word problems. Children with arithmetic learning disorders have particular difficulty in formulating and carrying out a cognitive plan which permits them to attack systematically a set of word problems. Based upon this observation, an instruction program utilizing instructor modeling of a verbally mediated systematic approach to word problem solving will be described.

Reporting Session V

ANALYSIS AND SYNTHESIS OF MATHEMATICS PROBLEM SOLVING PROCESSES

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Reactor: Philip Smith
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This analysis and critique session reports the results of a project which has had as its purpose the investigation of adolescents' problem solving processes. The existing conceptual framework upon which this project was developed is outlined in Goldin and McClintock (1980). The problem solving protocols of more than 500 students from sixth to ninth grade were obtained from over 20 researchers and anlyzed. The problem solving processes were then coded. At the same time, a comprehensive analysis was made of the problem tasks presented to the students. The process and task analysis were combined so that patterns in solution strategies could be studied across equivalent problems. Effects of different task structures on solution processes were also examined. Finally, the developmental trends in problem solving processes from sixth to ninth grade were studied. Equivalent problems solved by students at different grade levels were used to determine differences in the processes used by students as they mature and acquire more mathematics background.

The "think aloud" interview approach to research in problem solving is one of the primary methodolgies used to elucidate the cognitive and affective processes or strategies used by adolescents while solving mathematical problems. The advantages of this approach have been discussed previously (Hatfield, 1978). However, the interview approach has some limitations. The time required for an interview makes it difficult to study large numbers of subjects or problem tasks. Also there is a possibility for inconsistency both across interviews and among interviewers. These limitations affect the generalizability of results. Although a great deal of exploratory research remains to be done before clear theoretical models of mathematical problem solving processes can be produced, sufficient

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work has already been done to make some progress in synthesizing results. A synthesis will provide directions for further study and provide directions for the improvement of teaching problem solving. This project has begun the important task of analysis and synthesis of existing problem solving research necessary for accomplishing these goals.

The presentation will consist of descriptions of the four major components of the project, followed by a reaction by a member of the project Advisory Board.

Introduction: The rationale, objectives and procedures of the project will be presented. Data for the project have been obtained from the transcripts, audio-tapes, and problem sets made available by more than 20 researchers in mathematics education who had interviewed a diverse set of subjects in grades 6 - 9. The difficulties and subsequent procedures used in handling this type of data synthesis will be noted.

The coding analysis and the task variable analysis data will be made available to the mathematics education research community through the National Collection of Research Instruments for Mathematics Problem Solving. Kulm will describe the data to be made available through this source as well as outlining the plans for the dissemination of the results.

Process Coding Analysis: The analysis of a vast amount of process data from a diverse set of studies has posed some special problems. A coding system was needed that was efficient enough for use with hundreds of protocols yet sufficiently sensitive to reflect meaningful variations in processes. The coding system of Branca et al. (1980) served as the framework for analysis in the project. Reliability of coding by the four staff members was a major concern. Special data sampling techniques were used and training procedures were developed to assure that the staff maintained a high standard of coding reliability throughout the duration of the coding. Finally, procedures for recording unusual occurences and affective behavior were developed. Campbell will discuss these problems and their resolution. Emphasis will be placed on the development of the coding system and the statistical procedures for reliability assessment.

Task Variable Analysis: The detection of patterns in problem solving processes across tasks and the discovery of developmental trends depend on a careful task analysis. The task variable frameword described by Kulm (1980) was used to select variables in each of three categories: syntax, content/context, and structure. Special emphasis was placed on classifying problems whose structures were equivalent in order that solutions could be compared. Frank will describe the procedures used for task analysis and the selection of the variables. Difficulties encountered in analyzing the variety of problem tasks will be discussed. Procedures for analyzing and classifying problem structures will be a primary focus of this report.

Task-Process Relationships: A primary goal of the project was to synthesize the results of process analyses through a search for patterns. These patterns were found by considering solutions of well-described problems. Two main types of relationships between problems and processes were investigated. First, the effects of increased problem complexity on processing were studied. Do adolescents vary their strategies as the difficulty of the

problem changes? Second, the processes used by students of different ages on equivalent problems is important in assessing developmental patterns. Several sets of equivalent problem structures will be used to describe these trends from sixth to ninth grade. Talsma will also discuss implications for the teaching of problem solving, in view of the problem tasks presented to students.

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A STUDY OF YOUNG CHILDREN'S SPATIAL PREFERENCES WHEN COMPARING SELECTED TOPOLOTICAL AND GEOMETRIC PROPERTIES

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Purpose: There has been considerable research done with respect to children's spatial concepts. Much of the work has been based upon or related to Piaget's model of children's spatial development. Piaget's model states that children tend to develop topological notions (openness, closedness) first and then geometric equivalence (vertices, curvature, orientation). Thus, the representational space of children is predominatly topological early, with geometric notions developing later.

The research with respect to this theory is conflicting. Studies by Piaget and Inhelder (1963) and Thiron (1969) support these contentions while studies such as Martin (1976) and Geeslin and Shar (1979) give results contrary to Piaget's model. Martin's study indicated that geometric intuition is dominant at all age levels while Geeslin and Shar proposed that a child's preference depends on the amount of distortion in a figure.

The specific purposes of this study were:

1. To determine on which properties children focus when comparing spatial figures. The properties considered were orientation, curvature, openness, and connectedness.

2. To determine if children's focus on geometric or topological properties is developmental.

Procedure: Eighty-nine children, ages 4 through 8 (grades nursery through second) were used in this study. Each child was presented a series of 36 items designed to measure a child's spatial preferences when comparing figures. For example, a child was shown a model figure. The child was told to remember the figure because it was going to be removed. After this model was removed, two "copies" of the model were shown to the child.

For example, copy (a) was open where the model was closed while copy (b) differed from the model in orientation. The child was next asked to tell which of the copies was most like the original figure. A child who focused primarily on the orientation of figures would, theoretically, not choose the copy whose orientation was different from the model as being most like the model.

There were five different model figures with four variations of each and each variation differed from the model in one of the four properties already described. In responding to an item, a child was comparing each of two copies to the model figure and to each other. These properties were considered pairwise since a child might focus more on orientation only when compared to curvature.

After every sixth item, a child was asked why he/she had not chosen the other copy. These responses served as a check to make certain that the child was responding on the basis of the properties involved.

Each student received a score from 0 to 18 for each of the four attributes considered. For example, if a student always chose the copy that differed in orientation from the model as being most like the model, then an orientation score of 18 was recorded. Group (nursery, kindergarten, first grade, second grade) means were determined for each attribute score and compared to a theoretical (non-preferential) means of 9 using t-tests. A group orientation mean significantly lower than 9 indicated that the students would not see copies different in orientation as being most like the models; they preferred for orientation to remain invariant.

Group means were also calculated for each pairwise comparison. For example, since each child was asked six times to choose between a copy differing in orientation and a copy differing in curvature from the models, he/she received a orientation-curvature score from 0 through 6. The score referred to the number of times the child chose the copy differing in orientation (first attribute listed). T-tests were again used to compare group means to the theoretical mean of 3.

Finally, for the pairwise scores, a one-way analysis of variance was used to test for significant differences across grade levels. When significant F values were obtained, Duncan't Multiple Range test was used to test for pairwise differences between groups.

Results: Overall, at all grade levels, children tended to focus on the curvature of a figure when comparing to a model figure. This was evidenced by the fact that the subjects very seldom chose the copy differing in curvature as being most like the model. It was found that children at all age levels showed a strong focus (preference) on curvature of figures, regardless of the other attribute being varied. For most children, changing the curvature of a figure was perceived as making the figure unlike the original, at least in comparison to changing the openness, connectedness, or orientation. While there were several other significant pairwise means, there were no other overall preferences.

There were two significant results when pairwise means were compared across grade levels. For the open-orientation comparison ($F = 2.74$, p [illegible]), the kindergarten mean was significantly different from both the first and second grade means. First and second graders focused strongly on whether a copy was open or closed (as compared to the model). The kindergarten children did not have a preference for either attribute.

For the connected-orientation comparison ($F = 9.82$, $p < .001$), both the nursery and kindergarten means differed significantly from both the first and second grade means. In this instance, first and second graders focused strongly on connectedness. The nursery children showed no preference, while the kindergarten children actually focused more on the orientation of a figure; just the opposite of the first and second graders.

Conclusions: The results of this study did not support Piaget's model. For all age groups, the only predominant spatial focus was on the curvature of a figure. If young children develop topological awareness first, then the subjects would be expected to focus more on openness and connectedness.

Where focus on topological properties did exist, it was restricted to the older subjects. The results do not indicate that the younger children's representational space was predominatly topological. It is not clear whether the nature of the spatial properties or the amount of distortion was most influential on the children.

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Reporting Session VI

ASSESSING CHILDREN'S DEVELOPMENT IN GEOMETRY USING THE VAN HIELE LEVELS

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Purpose: The study provides an operational characterization of the lowest 4 van Hiele levels of development in geometry. The researchers used clinical interviews with children from grades 5 through 11 to develop a sequence of experimental tasks that are intended to place a child at a particular van Hiele level with respect ot a certain concept. The concepts considered in the present study are triangles, quadrilaterals, and polyhedra.

While the van Hiele levels have been extensively described in the setting of school mathematics (van Hiele 1959, 1973: Wirszup, 1976), very little work has been done in this country to provide a clincial description of the levels.

A characterization of the van Hiele levels may be very useful in assessing individual children's development in learning geometry, and in the development of geometry curricula for elementary and middle schools.

Procedure: Experimental tasks involving drawing, recognizing, sorting, defining, and anlyzing certain types of traingles, quadrilaterlas, and polyhedra have been developed. Clinical interviews, of about 3 hour duration (three one-hour settings) were used to present students the tasks in a fixed sequence. The students were questioned extensively by an interviewer (one of the researchers) about their specific responses to the tasks. The interviews were tape-recorded and analyzed by the team of 4 researchers. Each researcher evaluated the student's development on each of the three concepts.

Students were chosen to represent a variety of ages, abilities, and socio-economic classes. Approximately equal numbers of boys and girls were used. About 50 students were interviewed.

Analysis: Both qualitative and quantitative data have been gathered. Preliminary evaluation of the clinical interviews has provided evidence that suggests that the van Hiele levels do serve as a model of development in geometry. Strong qualitative evidence has been found which implies that student's reasoning about geometric concepts is initially based upon gross visual cues. Subsequently the reasoning process is selectively refined until abstraction and deduction are developed. These results appear to support the van Hieles' theory.

An attempt to correlate the interpretations of the interviews by the four researchers is currently in preliminary form. This correlation procedure will be further developed as the study proceeds from its Developmental Phase to the Experimental Phase. The correlation of different interviewer interpretations may provide some measure of inter-rater reliability for the clinical characterization of the van Hiele levels.

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Reporting Session VI

THE CHILD'S CONCEPT OF RATIO OF DISTANCES IN TWO DIMENSIONS

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Purpose: Effective placement of geometric concepts into a comprehensive school mathematics program requires a harmonious mesh of mathematical and cognitive structures. Felix Klein's Erlanger Programm provides the mathematical structure of geometric concepts. Jean Piaget's work provides a model of the child's cognitive structures. Both Kelin's Programm and Piaget's theory of cognitive development emphasize invariance through transformations. The ideas of Kelin and Piaget have been assimilated into a research model for the investigation of the child's construction of space (Martin, 1976). The model offers a hypothesized sequence for the order of the child's construction process.

Studies by mathematics educators of the child's concept of space have tended to focus on the endpoints in this sequence, i.e. topological space and Euclidean space. However, questions pertaining to other geometries in between these endpoints can be raised. Similarities form a subgroup of the affine group of transformations. Hence, if Kelin's Programm can be used as a model and if Piaget's suggested sequence is essentially correct, one would expect affine concepts to develop prior to similarity concepts. Affine transofrmations preserve ratios of distances along the same line or along parallel lines. Similarity transformations preserve ratios of distances in all directions. Some natural questions are: (1) When do children develop the ability to conserve ratios of distances in one direction? (2) Does this ability develop prior to the ability to conserve ratio in many directions?

In a previous study, Martin (1978) proposed 11 years of age as an approximate answer to the first question. One of the purposes of this investigation was to replicate Martin's study. The other purposes were to determine 1) whether the presence of a second dimension on essentially one dimensional tasks would serve as a distracter to those subjects who had been previously successful at conserving ratios of distances on one dimensional tasks, 2) whether conservation of ratios of distances in one dimension is necessary for conservation in two dimensions, and 3) when children develop the ability to conserve in two dimensions.

Procedure: The subjects of this clinical study were fourth, fifth and sixth graders who placed in the upper three quartiles on the nationally normed SRA test. Twenty fourth, 20 fifth and 20 sixth grade pupils were randomly selected to participate in the study. The mean ages by grade of the Ss were: fourth grade - 10 years, 2 months; fifth grade - 11 years, 2 months; sixth grade - 12 years, 1 month.

Phase One

Each subject was interviewed by one of the researchers and asked to complete nine one dimensional conservation tasks. The nine tasks essentially involved the placement of a bead on a wooden rod which was shorter (or longer) than a model which had a bead glued 5/7 of the length from one end. Ss were supplied with measuring aids of appropriate lengths and markings to permit bead placement without consciously quantifying a ratio. (See Martin, 1978, for a complete description of the materials and tasks.)

Phase Two

Twenty-three of the sixty Ss were successful on five or more of the nine tasks from interview one and classified as conservers of ratio in one dimension. These 23 Ss and 10 randomly selected Ss who had failed to reach the criterion level of five correct were interviewed a second time and asked to complete eight additional tasks. Two of the eight tasks were designed to test whether the addition of a second dimension distracted subjects in solving one dimension conservation of ratio tasks. The remaining six tasks were designed to test conservation of ratio in two dimensions. Four of these six tasks involved a change in size of a two dimensional configuration of sticks glued to a board. The final two tasks involved placing a dot in a rectangular field so that its position would reflect a size transformation of a model rectangular field containing a dot located in the upper right quadrant. For each task the Ss could use aids appropriate for determining length and/or position without consciously quantifying a ratio.

Results: Analysis of the results of the first interview, conservation of ratio in one dimension, involved computation of means by grade for the number of correct responses. An analysis of variance was computed to detect differences in performance by Ss at each grade level. The analysis showed a significant difference between the performance of sixth graders when compared to either fourth or fifth grade students. No significant difference was found between fourth and fifth graders. Post-hoc analysis of the mean age of conservers and non-conservers in one direction were computed using a t-test and showed a significant difference ($\alpha < .005$) ($\bar{X}_c$ = 11 years, 6 months, $\bar{X}_{nc}$ = 10 years, 10 months).

The results of the second interview were analyzed in three ways.
1) To determine whether a second deimension distracted subjects in solving one dimensional tasks, the correlation was computed between scored from the first interview and scores from the one dimensional measures on the second interview. Results by grade were: sixth grade,

r = .87; fifth grade, r = .97; and fourth grade, r = .22. 2) To determine whether differences existed between the scores (conservers vs. Non-conservers in one dimension) on the two dimensional tasks, a one tailed t-test was used. The analysis showed a significant difference ($\alpha < .025$) between the performance of the two groups. 3) To determine whether a difference existed between sixth grade conservers of ratio in one dimension and fifth grade conservers on the two dimension tasks, non-parametric techniques were used since cell sizes were small and unequal ($n_1 = 6$, $n_2 = 12$). The conservative Kolmogorov-Smirnov test showed no significant difference. Post-hoc analysis of the mean age of the conservers in two dimensions and the mean age of the non-conservers in two dimensions using a t-test showed a significant difference ($\alpha < .05$) in age.

Conclusions: From the results of the first interview we conclude that conservation of ratio in one dimension occurs between eleven and twelve years of age. This conclusion is consistent with the findings of Martin.

From the results of the second interview, there are three reasonable conclusions. 1) For students who conserve ratio of lengths in one dimension the addition of a second dimension does not serve as a distracter to their ability to conserve ratio in one dimension. 2) Children conserve ratio in one dimension before they conserve ratio in two dimensions. 3) Although we cannot expect the majority of sixth graders to conserve ratio in two dimension, some children at about twelve years of age begin conserve ratios in more than one dimension.

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Reporting Session VII

A STUDY FOR THE VALIDATION OF AN INSTRUCTIONAL SEQUENCE DESIGNED TO TEACH VERBAL PROBLEM SOLVING IN ELEMENTARY MATHEMATICS

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Purpose: This study used the Romberg-DeVault model to develop and revise an elementary mathematics instructional sequence based on the action-sequence approach to verbal problem solving. The instructional materials developed by this investigator required that in the process of solving one-step verbal problems the pupils would: 1) identify the set operation described in the verbal problem as joining, separating, or comparing; 2) select the arithmetic operation appropriate to this set operation; 3) select an open equation which included this arithmetic operation and described the action-sequence of the verbal problem. After learning and practice of these three steps, step three was replaced by 4) translating the action sequence of the verbal problem into an open equation.

Procedure: A pilot examination phase of the experiment consisted of pretest, eight consecutive fifty-five minute class periods of instruction and posttest. The study used two separate randomly selected samples. One sample consisted of thirty third-grade pupils using verbal problems solved by addition or subtraction. The other sample consisted of thirty fourth grade pupils using verbal problems solved by multiplication or division. The criterion for success on test items was producing an open equation appropriate for the solution of that item. The t-test for repeated measurement of the same subjects indicated no significant differences between pretest and posttest mean scores except for comparing-type verbal problems in which case posttest mean scores were higher at .05 level.

The hierarchical nature of the instructional sequence was examined by Guttman Scalogram Analysis. For the complete instructional sequence, the results did not support the hypothesis of linear hierarchy. For certain classes of verbal problems the results did support the hypothesis of linear hierarchy. The hierarchical nature of the instructional sequence was also examined by the Walbesser Hierarchical Validation Tests. For the complete instructional sequence the results did not support the hypothesis of linear hierarchy. For those classes of verbal problems for which result from Guttman Scalogram Analysis supported the hypothesis of linear hierarchy, the Walbesser Test also supported the hypothesis of linear hierarchy. Further for certain other classes of verbal problems, results from the Walbesser Test supported the hypothesis linear hierarchy.

Guttman Scalogram Analysis, being the more conservative test, was selected as the basis for revising the instructional sequence. The investigator developed a scheme of Minimal Sets for Error Covering as the criterion for selecting the instructional steps which were in need of revision. Results of this analysis indicated that the steps of the instructional sequence in need of revision were (1) and (3). Step three was replaced by (3') completing an open equation which described the action-sequence of the verbal problem. To overcome the errors associated with step one, the scoring of the posttest was revised to permit alternate interpretation of the action-sequence subject to the limitations: (a) the interpretation of the action-sequence was reasonable and (b) this alternate interpretation yielded a correct open equation. With respect to step one no alteration was made in the instructional sequence. The scoring of posttest revision was made so that after treatment no pupil was penalized for willfully choosing to use a previously learned but correct strategy for interpreting verbal problems.

A validation phase of the experiment using the revised instructional sequence of pretest, eleven consecutive thirty-five minute class periods of instruction, and posttest was conducted in another school system using randomly selected intact classes into which pupils had been randomly assigned. One sample consisted of two sections of third-grade pupils containing a total of fifty pupils. This sample used verbal problems solved by addition or subtraction. The other sample consisted of two sections of fourth-grade pupils containing a total of fifty pupils. This sample used verbal problems solved by multiplication or division. The t-test for repeated measurement of the same subjects indicated significant differences at the .01 level favoring the posttest mean scores.

Results: For the complete instructional sequence results from Guttman Scalogram Analysis did not support the hypothesis of linear hierarchy. However, for the instructional sequence using verbal problems solved by addition or subtraction, results from Guttman Scalogram Analysis did support the hypothesis of linear hierarchy. For the instructional sequence using verbal problems solved by multiplication or division the results indicated the proportion of scale-type response patterns to be .83, near the .85 required for validation.

The McNemar Test of Correlated Proportions indicated that the instructional sequence had no adverse effect upon pupils who, prior to treatment, were successful in the interpretation of verbal problems except for problems of an additive structure, joining-type verbal problems were incorrectly identified as comparing-type verbal problems. This confusion caused more than half of all errors of this problem type.

It was not planned that the verbal problem solving abilities of the pilot examination samples and the validation samples be equivalent as measured by the pretest. However, this did occur for third-grade pupils, permitting post-hoc analysis. The t-test for matched groups indicated no significant differences in effect of using an instructional sequence which was not hierarchically valid and an instructional sequence which was hierarchically valid. This result along with the result of the t-test for repeated measures together suggest the hypothesis that an hierarchically valid instructional sequence is a necessary but not a sufficient conditions for improving pupils' verbal problem solving ability.

Reporting Session VII

CALCULATOR USE AND PROBLEM SOLVING STRATEGIES OF GRADE SIX PUPILS

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Purpose: The purpose of this study was to determine the effects of problem-solving strategy training and calculator use. Additionally, the type and range of strategies used were studied with attention to the possible differential effect of problem-solving training and calculator use on pupils of three ability levels.

In recent years numerous studies have found that either an increase or no difference in pupil mathematics achievement results from calculator use in performing mathematics tasks. For example, Wheatley, Shumway, Coburn, Reys, Schoen, Wheatley, and White (1979) found that, over the short term, pupil performance on standardized mathematics tests do not differ when calculators are a part of instruction. However, few studies have investigated the effect of calculator use on problem-solving behavior. With increased emphasis on problem solving in the mathematics curriculum (NCTM, 1980) and the widespread availability of inexpensive calculators, it is important that the effect of calculator use in problem-solving strategy utilization be carefully studied. Zweng (1979), Blume (1979), and Wheatley (in press), using interview techniques, reported a positive effect of calculator use on the thought processes of students in problem solving. Shields (1980) argues that there will be less distraction in problem solving when calculators are used in computing and thus problem performance will be enhanced.

Procedure: The subjects for this study were 396 sixth-grade pupils in 18 intact classes drawn from a midwestern city of 50,000. The classes were randomly assigned to one of three treatment groups as shown below.

This is a report of a Two-year NSF-Rise Funded Project.

		Ability		
	Treatment	High	Middle	Low
I	Calculator Use and Problem Solving			
II	Problem Solving			
III	Control			

Tnree ability groups were formed using the scores on the Iowa Problem Solving Test (Schoen and Oehmke, 1980). This test was administered during the first week of school. The pupils in groups I and II studied specific problem solving techniques. The treatment materials were based, in part, on the work of Greenes, Immerzeel, Ockenga, Schulman, and Spungin (1980) and Schaaf (1979). Specific lessons were designed to teach five problem-solving strategies: guess and test, draw a diagram, simplify the problem, make a list, and find a pattern. Further, pupils were taught to use the four steps: understand the problem, make a plan, carry out the plan, and look back. The treatment consisted of 17 weeks of problem-solving training. The classes were taught by the classroom teacher using materials developed and supplied by the investigators. Each of five strategies were taught over a five-week period followed by lessons on decision making. The remaining weeks consisted of mixed practice on carefully selected problems. All pupil materials were provided to the teachers by the experimenters on a weekly basis.

Analysis: In February 1981 ninety pupils (30 from each treatment group) were interviewed using the think-aloud technique as they solved mathematics problems. In addition to statistical analysis of the data, the report will include case studies and anecdotal data. The meaning of statistical results may be greatly enhanced by other assessments.

During the interview each subject responded to six multi-step problems. For each subject, three problems were randomly selected to be solved without calculators, while calculators were available for the other three problems. The interview data were analyzed for strategy use, production, and computational errors.

A parallel form of the Iowa Problem Solving Test was administered during the eighteenth week. Using a 3 x 3 ANOVA, data from this instrument were used to determine the effects of treatment, ability, and interaction.

This report will present the results of group testing, pupil interviews, and case studies. While the data collection is not yet complete, presentation of results immediately upon completion of the study will provide researchers and teachers with "cutting-edge data."

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Reporting Session VII

EFFECTS OF PRIOR KNOWLEDGE AND CREATIVITY UPON PROBLEM SOLVING

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Purpose: Problem solving has long been identified as one of the most important goals of mathematics instruction. The National Council of Teachers of Mathematics in their recommendations for school mathematics in the 1980's has suggested that problem solving be the focus of mathematics instruction (1980). Various other groups over the years also have stressed the importance or problem solving in school mathematics programs (Cambridge Conference on School Mathematics, 1963; National Advisory Committee on Mathematics Education, 1975). The National Commission of Supervisors of Mathematics in their position paper listed problem solving as one of the ten basic mathematics skills and the principal reason for studying mathematics (1977). Thus, there is agreement on the importance of developing problem solving skills.

Although there is agreement on the importance of problem solving as an outcome of mathematics instruction, not enough is known about how to develop this skill in students. Results of the past two National Assessments (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1980) have indicated students are lacking in problem solving skills. Clearly more information is needed about the nature of problem solving in order to develop programs which will assist students in developing their problem solving abilities.

A variety of studies have focused on the effect of prior knowledge upon problem solving performance. It has been shown that reading level, mastery of computational skills, understanding of vocabulary, and knowledge of relevant mathematical concepts all correlate highly with success in problem solving (Aiken, 1971; Jerman, 1972; Smith, 1971; Chase, 1964; Alexander, 1960; VerderLinde, 1965). All of these studies emphasize that unless a student can read the problem, is familiar with the vocabulary contained in the problem, understands the posed question, and possesses the necessary computational skills, the student will have difficulty in solving the problem.

In addition to the effect of prior knowledge upon problem solving performance, other studies have examined traits of the problem solver including such variables as attitude, level of field independence, perseverence, and spatial ability, to name just a few. Long considered a correlate of problem solving is creativity or divergent thinking. It was the purpose of this study to examine the effects of both prior knowledge and creativity upon problem solving performance.

Procedure: Approximately 50 children in each of the grades two, four, and six participated in the study. The school selected for the study has children from low, middle, and upper income families and is located in a city of approximately 250,000 people.

Two instruments were used: Wearne-Romberg Problem Solving Test (Wearne, 1976) and Torrance Test of Creative Thinking (Torrance, 1966). The problem solving test is designed to yield an application (prior knowledge) and a problem solving score. A problem situation for this test was defined to be a situation which did not lend itself to an immediate application of some rule or algorithm. Three forms of the problem solving test were used, one at each grade level.

The Figural Form of the Torrance test was used in the second grade and the Verbal Form was used in the fourth and sixth grade. The creativity test produces four scores in the figural form and three for the verbal. The three scores common to both forms are fluency, flexibility, and originality with an elaboration score also included in the figural form results.

Results: The data were analyzed using multiple regression techniques with problem solving being considered the dependent variable and application, fluency, flexibility, originality, and elaboration all being independent variables. A series of nine multiple regressions were run, three at each grade level. The regression procedure, a stepwise regression process, first selected the best single predictor and then the best two predictors and continued in this fashion until all of the variables had been included in the model. The regression models finally selected were those whose predictors added a significant portion of predictive power to the model.

Prior knowledge was significantly correlated with problem solving with all of the groups considered with the exception of second grade girls. The relationship between creativity and problem solving is not as clearly defined. Although significant correlations existed between the measures of creativity and problem solving at grades two and six, it appears to be gender specific. None of the creativity scales were significantly correlated with problem solving for the boys at any grade level whereas two of the four scales were significantly correlated with problem solving for the second grade girls (fluency and originality) and all three scales were significantly correlated with problem solving for the sixth grade girls.

Conclusions: If one were to select the best single predictor of success in problem solving as measured in this study it would appear to be prior knowledge. The application score was more closely associated with problem solving than any of the creativity scales. However, for some of the older students it would appear that one can account for a significant portion of the variance remaining after the variance associated with application score is removed by considering the creativity measures; 65% of the variance in the problem solving scores for sixth grade girls can be accounted for by considering the application (prior knowledge) and creativity scales.

The study raises a number of questions regarding the relationship between divergent thinking and problem solving in mathematics as well as the ability to assess this trait.

Reporting Session VIII

AN ANALYSIS OF SELECTED COGNITIVE STYLE DIMENSIONS RELATED TO MATHEMATICS ACHIEVEMENT, APTITUDE, AND STTITUDES OF TWO-YEAR COLLEGE STUDENTS

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Purpose: The global intent of the research was to gain further insight into individual difference variables that affect the learning process. The specific purpose was to explore and examine the relationship of the correlative, predictive, and differential nature of selected cognitive style dimensions (conceptual tempo, field articulation, preceptive-receptive, intuitive-systematic, and locus of control) to the learning of mathematics at the two-year college level. The dimensions were analyzed to assess unique contributions to mathematics achievement, aptitude for mathematics, and attitudes toward mathematics. The influence of interrelationships among the dimensions on the mathematics criterion variables was examined, as well as possible confounding effects of intelligence.

Substantial findings exist relating cognitive styles to the learning and teaching of mathematics at various levels; and the implication is that cognitive styles have import at the college level. Yet, of over 250 references on cognitive style, relatively few studies using a college population were found.

Often the research techniques that were used severely limited both the validity and the generalizability of findings. Relatively few of the studies investigated examined the effects of more than one cognitive style dimension in relation to the learning of mathematics. Research findings tended to be broad and varied. The existent state of the art, then, is a body of literature--relatively sparse--from which it is difficult to extract general principles. Several researchers have pointed to the need for systematization at the levels of theory and research. Studies designed to relate the variables used by one investigator to those used by other investigators are needed.

The purposes of this research study were to extend present research findings to the two-year college level, to validate tentative findings from existing studies, and to test several raised questions. With the relative newness of cognitive style couples with the surfeit of existing cognitive style dimensions, the contention exists that more is to be

gained by broader unifying research as opposed to isolated investigations of only one or two dimensions to an overly specific aspect of learning mathematics. With the view of learning as possessing a multi-faceted nature, and with the plethora of multivariate techniques available, greater consideration can be given to the possible influence of cognitive style dimensions, not only in isolation, but in interactions with each other as they relate to the learning of mathematics.

A conceptual framework was built by overviewing cognitive style research, defining cognitive style, compiling known dimensions, citing existing and needed research, and showing the relevance of cognitive style research to the learning of mathematics.

Procedure: After a research model was presented, methodology and procedures were subdivided into eight major components: selection of cognitive style dimensions, pilot study, questions for investigation, instrumentation, population and sample, data collection, summary of hypotheses, and analysis of data.

Hypotheses were posed that focussed on the correlative, predictive, and differentiating aspects of cognitive style dimensions. Thirty students were randomly selected from each of six different sequences of mathematical study and were administered eleven assessment instruments within eleven weeks.

Analysis: Five subanalyses were used to examine the data: (1) Pearson product moment intercorrelations, (2) canonical correlational analysis between the two sets of variables, cognitive style dimensions and mathematics criterion variables, (3) multiple correlational analyses with the independent variables the cognitive style measures and the dependent variables, in turn, the three mathematics criterion measures, (4) multivariate analysis of variance with cognitive style measures as the independent variables, and (5) discriminant analyses.

Results: Pearson product moments revelaed several significant intercorrelations ($p < .001$) among the cognitive style dimensions. The systematic and intuitive measures were correlated, as were the preceptive and receptive measures. The systematic measure was correlated with field articulation. Latency and error scores on conceptual temp were negatively correlated. The intelligence measure correlated with field articulation, with the systematic measure, and negatively with conceptual tempo error scores. Intelligence also correlated with mathematics achievement and aptitude.

A canonical correlation of 0.66 was obtained between the set of cognitive style dimensions and the set of mathematics criterion variables. The systematic, intuitive, and field articulation measures from the first set were significantly related ($p < .001$) to aptitude and achievement.

Multiple correlations and commonality analysis showed the systematic and intuitive measures accounting for 14% of the variance in achievement. Of the variance in aptitude for mathematics, 41% was accounted for by the systematic, intuitive, and field articulation measures. The cognitive style

measures remaining significant contributors to the multiple R, when partialled of intelligence, were: intuitive measure for achievement, and systematic and intuitive measures for aptitude. Field articulation did not remain a significant contributor to aptitude. The multiple R between the cognitive style dimensions and attitudes was not significant.

An overall multivariate effect existed between the six groups of students on the cognitive style measures. Significant ANOVA's for four dimensions were followed by Tukey's post hoc comparisons. The dimensions: preceptive, receptive, field articulation, and conceptual tempo (mean latency) significantly contributed to group differentiation when partialled of intelligence.

From the profile of cognitive style dimensions, a discriminant analyses yielded two discriminant functions that differentiated students in the six sequences of mathematical study. Field articulation and the preceptive measure contributed to the discriminatory power of the first function; the systematic and receptive measures contributed to the second function.

The cognitive style dimensions were both interrelated, and differentially exhibited in the groups. Three dimensions were related to learning mathematics; the remaining dimensions may be related. Further research is needed on construct validity--especially for locus of control, preceptive-receptive, systematic-intuitive, and conceptual tempo.

For future research, recommendations were made for more thorough statistical analyses of the complex interactive arrangements among the cognitive style dimensions; the inclusion of both verbal and nonverbal measures of intelligence in cognitive style research; and the investigation of cognitive style related to more specific components of learning mathematics (e.g. aspects of problem solving, concept attainment strategies, etc.).

Reporting Session VIII

PROTOCOL ANALYSIS AS A TOOL FOR THE STUDY OF MATHEMATICAL PROBLEM SOLVING

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Purpose: Recent research in mathematical problem solving has recognized the need to conduct qualitative analyses of problem solving behavior. In order to do this, instruments must be developed which are capable of detecting the processes which subjects use as they solve problems. The present study was designed to describe the problem solving processes used by high school students as they attempted to solve non-trivial mathematics problems in a geometry course. The primary goals were as follows: 1) to observe and record subjects' problem solving behavior; 2) to refine and apply protocol analysis instrument for interpreting problem solving behavior; 3) to describe and interpret regularities in problem solving behavior so that inferences can be made concerning the planning strategies and heuristic methods used for solution.

Information processing theories have recently been applied to the study of problem solving to describe how persons solve problems which require planning and strategy. Problem solving is viewed both as an information gathering process and as a search process. Information contained in one knowledge state is used to guide the generation of new krowledge states. In solving a problem, a subject constructs a path through a space of knowledge states until a state is reached which provides a solution to the problem.

General heuristics of value in mathematical problem solving have been identified are fairly well recognized (Poly, 1957; Lucas, 1977; Schoenfeld, 1980). Specific heuristics are those content dependent rules, tricks and short-cuts which enable a problem solver to effectively use specific content knowledge when it might prove to be useful. How well a subject is able to solve problems in a given content area of mathematics depends, among other things, on how well he understands the subject matter and on how effectively he can apply content specific and/or general heuristics in the search for a solution. The study of mathematical problem solving can be viewed as the study of the systems of heuristics which subjects use as they attempt to exploit the information in a task environment.

Procedure: The research was clinical and descriptive, rather than experimental. Fourteen subjects of various mathematical abilities were individually presented with eight problems, approximately one per week. Subjects were asked to think aloud as they attempted to solve each problem. Their verbal statements were tape recorded, and detailed notes were kept of written work. Audiotapes were transcribed, and written figures were displayed and cross-referenced to the transcripts of verbal statements.

Transcripts were coded by assigning one code symbol to each phrase representing a piece of problem information or the use of an operator. The coding system used was similar to those used by Kantowski (1974) and Lucas (1977). Problem behavior graphs similar to those used by Newell and Simon (1972) were drawn to further trace the subject's attempted solution path. Knowledge states and operators used by the subject were infered from the coded protocol and were represented graphically to reconstruct a history of the search process.

Results: Preliminary results indicate that a high degree of reliability can be obtained in the use of the coding and graphing instrument, and that these instruments can be effectively used to interpret subjects' problem solving behavior. The general strategy of means-ends analysis, described by Greeno (1978) and Simon (1978), was used by a number of the subjects on some of the problems. The processes of identifying subgoals and difference-reducing operators was not evident, however, across all individuals and all problems. A more common strategy was the exploration and development of given data and an opportunistic approach to planning. Three variables which seem to affect the way in which subjects searched for solutions and planned solution strategies are as follows: problem characteristics, individual differences, and level of expertise in the subject matter.

Conclusions: One of the perplexing questions confronting the classroom teacher is why some students are consistently unable to succeed on problems for which they have prerequisite knowledge and skills. The results of this study suggest that most students require instruction in the use of planning strategies and heuristic methods. Further, students of different mathematical abilities may benefit from different kinds and amounts of process-oriented instruction.

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Reporting Session VIII

THE EFFECTS OF EXTERNAL STRUCTURE AND COGNITIVE STYLE ON LEARNING MODULO FIELDS

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Purpose: Rather than investigating the best instructional technique for all learners, recent educational research has focused on the interaction between learner aptitudes and instructional treatments. In mathematical aptitude-treatment interaction (ATI) studies, one of the aptitudes that has been found to interact with a variety of instructional modes is general reasoning ability.

The search for other general aptitudes to use in differentially predicting student learning has lead other researchers to investigate the cognitive style of field-dependence-independence (F-D-I) in ATI studies. In a study by Renzi (1974), reported in Witkin, H.A. et al (1977), F-D-I interacted with the amount of feedback provided by the treatments. "...field-dependent students performed significantly better on the posttest when feedback was provided in the text" (Witkin, 1977, p.23).

Other studies reported by Witkin (1977)indicate that the amount of structure in the material also interacts with F-D-I. Field-dependent learners may be handicapped in their learning of material that lacks inherent structure and organization. Field-dependent learners may need more explicit knowledge of the goals of study and how the new material to be learned is related to their existing knowledge.

But it has also been found that field-independent persons tend to favor abstract and less personal subjects such as mathematics. (Witkin, H.A. et. al. (1977) p. 12-13. The interaction between cognitive style and instructional variables may therefore depend not only on the learner's cognitive style, but on the subject matter as well. Witkin, H.A. (1976) states:

> "Still to be considered in its implication for the classroom is the effect of the subject matter being studied on teacher-student interaction as a function of their cognitive styles. Take mathematics, for example. In view of the clear linkage between field independence and competence in mathematics would match or mismatch have different effects, if mathematics is the subject area rather than, for example, the social sciences?" (p. 65)

The purpose of this study was to search for interactions of general reasoning ability and field-dependence-independence with two mathematics instruction treatments. The objective is to provide evidence for or against the above question posed by Witkin.

Procedure: One hundred and four students from four sophomore level high school mathematics classes participated in the study. Complete data was obtained for 100 of these students.

General reasoning ability, measured by the Necessary Arithmetic Operations (NAO) test, and F-D-I, measured by the Hidden Figures Test: Cf-1 (HF), were assessed during one class period. Subjects were randomly assigned to instructional treatments which they studied for the next two periods. Achievement and transfer tests were given the fourth day; followed a month later with parallel retention tests.

Two instructional treatments on the topic of modulo systems as fields were developed. The treaments differed only in the inclusion or exclusion of three 'external structuring' variables. The structured treatment contained feedback of the correct answer for every exercise, explicit objective statements for each section of the treatments, and an introductory section on the properties of the real numbers. The other treatment, called the unstructured treatment, was exactly the same except that the feedback, objectives, and introductory section were excluded.

Four 15-item posttests were developed to measure achievement (AC), transfer (T), achievement-retention (RA), and transfer-retention (RT).

The hypotheses were tested using multiple linear regression model. The alpha level of rejection was .05.

Results: General Reasoning, as measured by NAO, was a significant predictor for the structured group for every dependent measure. This result agrees with the findings of Eastman and Carry (1975), Eastman and Beher (1975), McLeod and Adams (1980) and McLeod and Briggs (1980) and continues to indicate that NAO is a stable predictor of structured (deductive) learning situations.

There were no interactions of F-D-I with any of the posttests; achievement (AC), transfer (t), achievement retention (RAC), or transfer retention (RT) Ruthermore, F-D-I was not a significant predictor for either of the groups on any posttest. These results are consistent with the results of McLeod and Adams (1980) and also agree with and support the findings in McLeon and Briggs (1980) for three of the four dependent variables used. They found no interactions for F-D-I with level of guidance (external structure) for tests for tests of immediate achievement, achievement retention, and transfer retention. McLeod and Briggs (1980) suggest that the reason for the lack of interaction is that "...both treatments were highly structured." which "...probably reduced the likelihood of finding a consistent pattern of interactions with field independence."

Conclusions: The results of this study concerning mathematical subject matter, and the results from the literature applying to social science subject matter, imply that the natural structuring of mathematical material may provide all of the structure that is necessary for field dependent subjects to learn well. Additional external structuring of the subject matter such as provided in this study by feedback, knowledge of objectives and an introductory section, provide no additional help to field dependent subjects other than what is inherently supplied by the mathematics itself. The cognitive style variable of F-D-I, which has a well-documented history of success in predicting learning in unstructured social science settings, was unsuccessful in predicting learning of mathematical subject material. Witkin's (1976) statement about the effect of the subject matter on the interaction between cognitive style and learning (quoted in the introduction), which implied that mathematical subject material may indeed produce different results with respect to F-D-I than social science material, is supported by these findings. Witkin's question taken together with the results of the McLeod studies and this study suggest and support the following hypothesis:

> When mathematical material is the subject area, the interaction between field-dependence-independence and the external structuring of the subject material will be different than if the subject area is, for example, the social sciences. Furthermore, the inherent structure of mathematical material is so strong, that it precludes field-dependence-independence from being a differential predictor of achievement when using external structure as a treatment variable.

This is in direct contrast to the research findings with subject matter in the social sciences where it has been documented that field-dependence-independence is a differential predictor of achievement.

It is true that one cannot "prove" a negative hypothesis. However, when sufficient evidence suggests a direct deviation from the established research findings for a particular subject area such as mathematics, it becomes increasingly important for the mathematics education research community to examine the findings carefully and to base their future research on these findings.

These results, of course, need replication before generalization to all mathematical subject material can be made. But the primary indications are that the inherent structure in mathematical subject matter causes it to behave significantly different with field-dependence-independence from the results in non-mathematical subject matter.

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